

SCALAR RADIUS OF THE PION AND TWO PHOTONS INTO TWO PIONS. STRONG S-WAVE FINAL STATE INTERACTIONS

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Abstract

The quadratic pion scalar radius, $\langle r^2 \rangle_s^\pi$, plays an important role for present precise determinations of $\pi\pi$ scattering. The solution of the Muskhelishvili-Omnès equations for the non-strange null isospin (I) pion scalar form factor determines that $\langle r^2 \rangle_s^\pi = 0.61 \pm 0.04 \text{ fm}^2$. However, by using an Omnès representation of this form factor, Ynduráin recently obtains $\langle r^2 \rangle_s^\pi = 0.75 \pm 0.07 \text{ fm}^2$. A large discrepancy between both values, given the precision, then results. We show that Ynduráin's method is indeed compatible with the determinations from the Muskhelishvili-Omnès equations once a zero in the scalar form factor for some S-wave $I = 0$ T -matrices is considered. Once this is accounted for, the resulting value is $\langle r^2 \rangle_s^\pi = 0.63 \pm 0.05 \text{ fm}^2$.

On the other hand, we perform a theoretical study of the reaction $\gamma\gamma \rightarrow \pi^0\pi^0$ based on dispersion relations. The large source of uncertainty for $\sqrt{s} \gtrsim 0.5 \text{ GeV}$, due to variations in the phase used in the Omnès function above the $K\bar{K}$ threshold, is removed by taking one more subtraction in the dispersion relation. This allows us to make sharper predictions for the cross section so that one could use this reaction to distinguish between different low energy $\pi\pi$ parameterizations, once independent experiments are available. We also study the role played by the σ or $f_0(600)$ meson in this reaction and determine its width to two photons.

1 Introduction

Here we summarize the two papers [1,2] that mainly handle with the strong influence of the $I = 0$ S-wave meson-meson final state interactions. We concentrate here on the non-strange $I = 0$ scalar form factor of the pion [1] and $\gamma\gamma \rightarrow \pi^0\pi^0$ [2]. Both processes can be formulated in a way that has in common the same basic function in order to take care of the strong final state interactions in the $I = 0$ S-wave. This function has been recently the origin of large uncertainties in its implementation in the literature, both for the scalar form factor of the pion [3–5] and for $\gamma\gamma \rightarrow \pi^0\pi^0$ [6].

The scalar form factor of the pion, $\Gamma_\pi(t)$, corresponds to the matrix element

$$\Gamma_\pi(t) = \int d^4x e^{-i(q'-q)x} \langle \pi(q') | (m_u \bar{u}(x)u(x) + m_d \bar{d}(x)d(x)) | \pi(q) \rangle, \quad t = (q'-q)^2. \quad (1)$$

Performing a Taylor expansion around $t = 0$,

$$\Gamma_\pi(t) = \Gamma_\pi(0) \left\{ 1 + \frac{1}{6} t \langle r^2 \rangle_s^\pi + \mathcal{O}(t^2) \right\}, \quad (2)$$

where $\langle r^2 \rangle_s^\pi$ is the quadratic scalar radius of the pion. The quantity $\langle r^2 \rangle_s^\pi$ contributes around 10% to the values of the S-wave $\pi\pi$ scattering lengths $a_0^0 = 0.220 \pm 0.005 M_\pi^{-1}$ and $a_0^2 = -0.0444 \pm 0.0010 M_\pi^{-1}$, as determined in Ref. [7] by solving the Roy equations with constraints from two loop Chiral Perturbation Theory (CHPT). If one takes into account that one has a precision of 2.2% in the scattering lengths, a 10% of contribution from $\langle r^2 \rangle_s^\pi$ is a large one. Related to that, $\langle r^2 \rangle_s^\pi$ is also important in $SU(2) \times SU(2)$ CHPT since it gives the low energy constant $\bar{\ell}_4$ that controls the departure of F_π from its value in the chiral limit [8,9] at next-to-leading order.

Based on one loop χPT , Gasser and Leutwyler [8] obtained $\langle r^2 \rangle_s^\pi = 0.55 \pm 0.15 \text{ fm}^2$. This calculation was improved later on by the same authors together with Donoghue [10], who solved the corresponding Muskhelishvili-Omnès equations with the coupled channels of $\pi\pi$ and $K\bar{K}$. The update of this calculation, performed in Ref. [7], gives $\langle r^2 \rangle_s^\pi = 0.61 \pm 0.04 \text{ fm}^2$. Mousallam [11] employs the same approach and obtains values in agreement with the previous result. One should notice that solutions of the Muskhelishvili-Omnès equations for the scalar form factor rely on non-measured T -matrix elements or on assumptions about which are the channels that matter. Other independent approaches are then most welcome. In this respect we quote the works [12–14], and Ynduráin's ones [3–5]. These latter works have challenged the previous value for $\langle r^2 \rangle_s^\pi$, shifting it to the larger $\langle r^2 \rangle_s^\pi = 0.75 \pm 0.07 \text{ fm}^2$.

If this is translated to the scattering lengths above, employing an equation of Ref. [7], it implies a shift of $+0.006 M_\pi^{-1}$ for a_0^0 and $-0.001 M_\pi^{-1}$ in a_0^2 . Thus, one is referring to a shift of slightly more than one sigma. Refs. [3,4] emphasize that one should have a precise knowledge of the $I = 0$ S-wave phase shifts, $\delta_0(s)$, for $s \geq 4M_K^2$ GeV², M_K is the kaon mass, to disentangle which of the values, either that of Ref. [7] or [3], is the right one. However, this point is based on an unstable behaviour of the solution of Ref. [3] with respect to the value of $\delta_0(4M_K^2)$. Once this instability is cured, as shown below, the resulting $\langle r^2 \rangle_s^\pi$ only depends weakly on $\delta_0(s)$, $s \geq 4M_K^2$, and is compatible with the value of Ref. [7].

Regarding the reaction $\gamma\gamma \rightarrow \pi^0\pi^0$ one has to emphasize that due to the absence of the Born term (as the π^0 is neutral), this reaction is specially sensitive to final state interactions. For energies below 0.6 GeV or so, only the S-waves matter, which have $I = 0$ or 2. It is in this point where both the study of this reaction and the scalar form factor match. Recently, Ref. [6] updated the dispersive approach of Ref. [15] to calculate $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$. Here one finds a large uncertainty in the results for $\sqrt{s} \geq 0.5$ GeV that at around 0.6 GeV is already almost 200%. Again, this is due to the lack of a precise knowledge of the phase of the $\gamma\gamma \rightarrow \pi\pi$ $I = 0$ S-wave amplitude above $4m_K^2$.

We showed in Refs. [1,2] that one can improve largely this situation by employing an appropriate Omnès function in the $I = 0$ S-wave. The key point is that this function should be continuous under changes in the phase functions used above 1 GeV, a point overlooked in the previous studies.

2 The scalar form factor

Ref. [3] makes use of an Omnès representation for the pion scalar form factor,

$$\Gamma_\pi(t) = P(t) \exp \left[\frac{t}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\phi_0(s')}{s'(s' - t - i\epsilon)} \right]. \quad (3)$$

Here, $P(t)$ is a polynomial in t normalized such that $P(0) = \Gamma_\pi(0)$ and whose zeroes are those of $\Gamma_\pi(t)$. On the other hand, $\phi_0(t)$ is the continuous phase of $\Gamma_\pi(t)/P(t)$. Then Refs. [3,4] make use of asymptotic QCD which predicts that the scalar form factors should go as $-1/t$ times a positive smooth factor for $t \rightarrow +\infty$, so that the phase of the form factor should tend to $+\pi$ in the same limit. At this point, Refs. [3,4] make an assumption that is not always necessarily fulfilled. Namely, to identify $\phi_0(t)$ with the phase of $\Gamma_\pi(t)$, that we denote in the following as $\rho(t)$. If this identification is done, as in Refs. [3,4], it follows that $P(t)$ must be a constant, $\Gamma_\pi(0)$, because the behaviour for

$t \rightarrow +\infty$ that follows from Eq. (3) is

$$\Gamma_\pi(t) \rightarrow (-1)^{-\phi(\infty)/\pi} t^n t^{-\phi(\infty)/\pi} \Gamma_\pi(0) , \quad (4)$$

with n the degree of $P(t)$. As QCD implies in this assumption that $\phi(\infty)/\pi = 1$, then $n = 0$ and hence $P(t) = \Gamma_\pi(0)$, just a constant. One must be aware that in Eq. (3) $\phi_0(t)$ is the phase of $\Gamma_\pi(t)/P(t)$. Notice that the phase of $\Gamma_\pi(t)$ is not continuous when crossing a zero located at $t_1 \in \mathbb{R}$, as there is a flip in the sign when passing through. However, the phase of $\Gamma_\pi(t)/P(t)$ is continuous, since the zero is removed. This is the phase one should use in the Omnès representation, Eq. (3), because it results from a dispersion relation of $\log \Gamma_\pi(t)/P(t)$, and then $\phi(t)$ must be continuous (but not necessarily $\rho(t)$).

As stated, Ref. [3] took

$$\Gamma_\pi(t) = \Gamma_\pi(0) \exp \left[\frac{t}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\rho(s')}{s'(s' - t - i\epsilon)} \right] . \quad (5)$$

So that the scalar form factor is given by,

$$\langle r^2 \rangle_s^\pi = \frac{6}{\pi} \int_{4M_\pi^2}^{+\infty} \frac{\rho(s)}{s^2} ds . \quad (6)$$

The phase $\rho(s)$ is fixed in Refs. [3, 4] by invoking Watson's final state theorem. For $s < s_K$, $s_K = 4M_K^2$, it implies that $\rho(s) = \delta_0(s)$, where neglecting inelasticity due to multipion states, an experimental fact. For $1.42 > \sqrt{s} \gtrsim 1.1$ GeV, Ref. [3] stressed the interesting fact that experimentally the inelasticity turns out to be small and hence Watson's final state theorem can be applied approximately again. In the narrow region between $2M_K$ and 1.1 GeV inelasticity cannot be neglected but Ref. [3] argues that, as it is so narrow, its contribution to Eq. (6) is small anyhow and, furthermore, that the elasticity parameter η is not so small, so that one could still apply Watson's final state theorem with corrections. Thus, for $s_K < s < 2$ GeV², Ref. [3] identifies again $\rho(s) \simeq \delta_0(s)$. Finally, for $s > s_0 = 2$ GeV² Ref. [3] takes a linear extrapolation from $\delta_0(s_0)$ to π . One should here criticize that it is still a long way to run from values of $\delta_0(s_0) \lesssim 2\pi$ up to π at $s \rightarrow +\infty$. With all these ingredients, and some error estimates, the value $\langle r^2 \rangle_s^\pi = 0.75 \pm 0.07$ fm² results [3, 4].

As discussed above in the lines of Ref. [1], the steps performed in Ref. [3] are not always compatible. In Ref. [1] we took as granted the assumption that Watson's final state theorem can be approximately applied for 1.5 GeV $> \sqrt{s} > 2M_K$. Our assumption is in agreement with any explicit calculation of the pion non-strange $I = 0$ scalar form factor [7, 10, 11, 13] and it is the proper

generalized version of the assumption of Refs. [3,4] of identifying $\rho(s) \simeq \delta_0(s)$. Now, Watson's final state theorem implies that $\phi(s) = \varphi(s)$ (modulo π), with $\varphi(s)$ the phase of the $I = 0$ S-wave $\pi\pi$ amplitude, $t_{\pi\pi} = (\eta e^{2i\delta_0} - 1)/2i$. It occurs, as stressed in Refs. [4,16], that $\varphi(s)$ can be either $\sim \delta_0(s)$ or $\sim \delta_0(s) - \pi$ depending on whether $\delta_0(s_K) > \pi$ or $< \pi$, respectively, for $s_K < s < 2 \text{ GeV}^2$. The latter case corresponds to the calculation in Ref. [7], while the former is the preferred one in Ref. [4] and arguments are put forward for this preference in this reference.

Let us evolve continuously from one situation ($\delta_0(s_K) < \pi$) to the other ($\delta_0(s_K) > \pi$). In the first case $\varphi(s)$ has an abrupt drop for $s > s_K$ simply because then $\eta < 1$ and while the real part of $t_{\pi\pi}$ rapidly changes sign, its imaginary part is positive (> 0). The rapid movement in the real part is due to the swift one in $\delta_0(s)$ in the $K\bar{K}$ threshold due to the $f_0(980)$ resonance. As a result for $s \lesssim s_K$, $\varphi(s) = \delta_0(s) \simeq \pi$ and for $s \gtrsim s_K$ then $\varphi(s) < \pi/2$. This rapid movement gives rise to a rapid drop in the Omnès function, Eq. (5), so that the modulus of the form factor has a deep minimum around s_K . Here, one is using Watson's final state theorem with $\phi_0(s) = \varphi(s)$ and the form factor of Ref. [10] is reproduced. Notice as well that in this case the function $\phi(s)$ approaches π from below for asymptotic s and then $P(t) = \Gamma_0(0)$ in Eq. (3). Now, we consider the limit $\delta_0(s) \rightarrow \pi^-$ for $s \rightarrow s_K^-$. The superscript $- (+)$ indicates that the limit is approached from below (above). In the limit, the change in sign in the real part of $t_{\pi\pi}$ occurs precisely at s_K , so that for $s = s_K^-$, $\varphi(s) = \pi$ and for $s = s_K^+$ then $\varphi(s) < \pi/2$ (indeed it can be shown from unitarity that must be 0). As a result one has a drop by $-\pi$ in $\varphi(s)$ which gives rise to a zero in the Omnès representation of the scalar form factor. Thus, the deep has evolved to a zero when $\delta_0(s_K) \rightarrow \pi^-$. Because of this zero the proper Omnès representation now involves a $P(t) = \Gamma_\pi(0)(1 - t/s_K)$ and $\phi(s)$ is no longer $\varphi(s)$ but $\simeq \varphi(s) + \pi \simeq \delta_0(s)$ for $2.25 \text{ GeV}^2 > s > s_K$. This follows simply because $\phi(s)$ is continuous. Thus, we go into a new realm where $\phi(s) \simeq \delta_0(s)$ and the degree of $P(t)$ is 1, so that $\Gamma_\pi(t)$ has a zero at the point s_1 where $\delta_0(s_1) = \pi$ and $s_1 < s_K$. Note that only at s_1 the imaginary part of $\Gamma_\pi(t)$ is zero and this fixes the position of the zero [1]. We should emphasize here that if one uses Eq. (5) with $\phi(s) \simeq \delta_0(s)$, as in Refs. [3,4], then in the limit $\delta_0(s) \rightarrow \pi^+$ for $s \rightarrow s_K^+$ the Omnès representation would give rise to $|\Gamma_\pi(s_K)| = \infty$, while in the previously discussed limit of $\delta_0(s) \rightarrow \pi^-$ for $s \rightarrow s_K^-$ one has $|\Gamma_\pi(s_K)| = 0$. This discontinuity was corrected in Ref. [1] and it is the benchmark for a jump by one unit in the degree of $P(t)$, a discrete function, in Eq. (3).

Hence for $\delta_0(s_K) \geq \pi$ one has to use

$$\Gamma_\pi(t) = \Gamma_\pi(0) \left(1 - \frac{t}{s_K}\right) \exp \left[\frac{t}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\phi(s')}{s'(s' - t - i\epsilon)} \right], \quad (7)$$

with $\phi(s) \simeq \delta_0(s)$ for $s < 2.25 \text{ GeV}^2$. The uncertainties in this approximation for $s > s_K$ are discussed in Ref. [1] and included in the final error in $\langle r^2 \rangle_s^\pi$. The estimation is based in diagonalizing the $I = 0$ S-wave S-matrix for $s < 2.25 \text{ GeV}^2$, so that two elastic channels can be singled out [4]. We also remark that now $\phi(s)$ for $\delta_0(s_K) \geq \pi$ must tend to 2π asymptotically so as to match with the asymptotic behaviour of $\Gamma_\pi(t)$ as $-1/t$. In this way we have now a very soft matching with asymptotic QCD since for s around 2.25 GeV^2 , $\delta_0(s) \simeq 2\pi$. This was not the case in Ref. [3,4]. Notice that from our work it follows that the precise knowledge of the asymptotic behaviour of the phase of the form factor is not relevant as $\phi(s)$ can tend either to 2π ($\delta_0(s_K) > \pi$) or to π ($\delta_0(s_K) < \pi$), and the results are very similar.

Our final value is

$$\langle r^2 \rangle_s^\pi = 0.63 \pm 0.05 \text{ fm}^2. \quad (8)$$

The error takes into account different $\pi\pi$ $I = 0$ S-wave parameterizations, namely those of Refs. [7] and [17], the error in the application of Watson's final state theorem above 1 GeV and up to 1.5 GeV, and the uncertainties in $\phi(s)$ given by asymptotic QCD for $s > 2.25 \text{ GeV}^2$. This value is compatible with that of Ref. [7], $\langle r^2 \rangle_s^\pi = 0.61 \pm 0.04 \text{ fm}^2$, and also with $\langle r^2 \rangle_s^\pi = 0.64 \pm 0.06 \text{ fm}^2$ of Ref. [13] calculated from Unitary CHPT.

3 The $\gamma\gamma \rightarrow \pi^0\pi^0$ reaction

In this section we briefly review Ref. [2]. This reference extended the approach of Refs. [6,15] so as to be less sensitive to the phase of the $I = 0$ S-wave $\gamma\gamma \rightarrow \pi\pi$ amplitude above s_K . For this phase one has a similar situation to that of the scalar form factor of the pion, it can be either $\sim \delta_0(s)$ or $\sim \delta_0(s) - \pi$ for $1 \lesssim s \lesssim 2.25 \text{ GeV}^2$ [2,6]. In the approach of Ref. [6] this originates an uncertainty that raises dramatically with energy above 0.5 GeV, such for $\sqrt{s} \simeq 0.6 \text{ GeV}$ it is already 200%.

Let us denote by $F_I(s)$ the S-wave $I = 0$ $\gamma\gamma \rightarrow \pi\pi$ amplitude. The approach of Ref. [6,15] is based on isolating the left hand cut contribution of F_I which is denoted by L_I . These authors also employ the Omnès function

$$\Omega_I(s) = \exp \left[\frac{s}{\pi} \int_{4M_\pi^2}^{+\infty} ds' \frac{\phi_I(s')}{s'(s' - s)} \right], \quad (9)$$

where $\phi_I(s')$ is the phase of $F_I(s)$. For $I = 2$ by the application of Watson's final state theorem one has that $\phi_2(s) = \delta_2(s)$. For $I = 0$ and $s < s_K$, $\phi_0(s) = \delta_0(s)$. In the interval $1.5 > \sqrt{s} > 1.1$ GeV, $\phi_0 = \delta_0$ (modulo π) because inelasticity is small again, as already remarked. Similarly as in the scalar form factor one can have because of the onset of inelasticity above $2M_K$ and up to 1.1 GeV, that ϕ_0 is given either by $\sim \delta_0$ or $\sim \delta_0 - \pi$.

Ref. [6] then performed a twice subtracted dispersion relation of the function $(F_I(s) - L_I(s))/\Omega_I(s)$. An important point to realize is that the previous function has no left hand cut and that F_I/Ω_I has no right hand cut. Making use of the Low's theorem, which implies that $L_I(s)$ is given by the Born term $B_I(s)$ for $s \rightarrow 0$, one is only left with two subtraction constants to be fixed. One of these constants can be fixed by requiring that the $\gamma\gamma \rightarrow \pi^0\pi^0$ S-wave amplitude, $F_N(s)$, has an Adler zero around M_π^2 . The other one was fixed in Ref. [6] by requiring that the $\gamma\gamma \rightarrow \pi^+\pi^-$ S-wave amplitude, $F_C(s)$, tends to the Born term $B_C(s)$ for $s \rightarrow 0$ up to $\mathcal{O}(s^2)$. One has to say that Ref. [6] did not include axial vector exchanges which indeed give rise to a term that vanishes for $s \rightarrow 0$ only linearly in s . This gives rise to a difference in the cross section of around a 30% at $\sqrt{s} \simeq 0.5$ GeV.

In order to better handle the ambiguities in $\phi_0(s)$ above 1 GeV, Ref. [2] only uses $\Omega_0(s)$ of Eq. (9)¹ for $\phi_0(s) \sim \delta_0(s) - \pi$ for $s > 1$ GeV². For the case $\phi_0(s) \sim \delta_0(s)$ above 1 GeV Ref. [2] employs

$$\tilde{\Omega}_0(s) = \left(1 - \frac{s}{s_1}\right) \exp \left[\frac{s}{\pi} \int_{4M_\pi^2}^{+\infty} ds' \frac{\phi_I(s')}{s'(s' - s)} \right], \quad (10)$$

and then a twice dispersion relation of $(F_0 - L_0)/\tilde{\Omega}_0$ is performed. It is important to realize, as stressed in Ref. [2], that because of the first order polynomial in front of the exponential in Eq. (10), one indeed has a *three* times subtracted dispersion relation for $(F_0 - L_0)/\Omega_0$. Recall that the latter is the original function used in Refs. [6, 15].

Because of this extra subtraction one can reduce dramatically the sensitivity to the $\phi_0(s)$ above 1 GeV. The conditions used to fix the at most three subtraction constants that appear in our scheme are: i) $F_N(s) \rightarrow 0$ for $s \rightarrow 0$ with the slope fixed by one loop CHPT [18] (with an uncertainty of around 15%), ii) $F_C(s) \rightarrow B_C(s) + \mathcal{O}(s)$ with the rest fixed by one loop CHPT (with the same 15% of estimated uncertainty). The third condition is an upper bound to the value of the resulting cross section in the $f_0(980)$ region so that it is smaller than 200 nb. Notice that its experimental value is smaller than 40 nb and, hence, we take here a very conservative uncertainty.

¹We already know about the lack of continuity of $\Omega_0(s)$ when $\delta_\pi(s_K)$ crosses π when taking $\phi_0(s)$ given by $\varphi(s)$ as in the case of the scalar form factor.

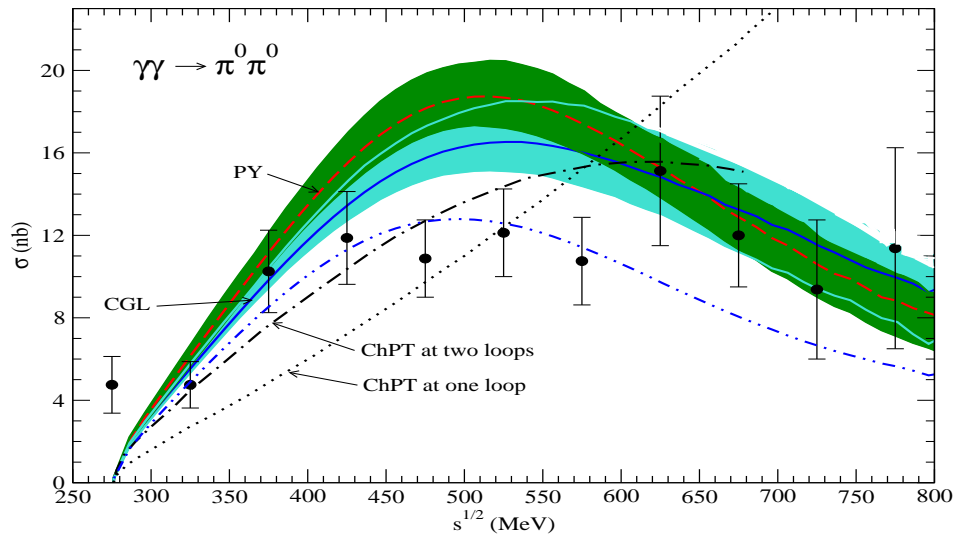


Figure 1: Final results for the $\gamma\gamma \rightarrow \pi^0\pi^0$ cross section. Experimental data are from the Crystal Ball Coll. [19], scaled by $1/0.8$, as $|\cos\theta| < 0.8$ is measured and S-wave dominates. The lighter band corresponds to Ref. [7] while the darker one to Ref. [17]. The dot-dot-dashed line results after removing the axial vector exchange contributions, as in Ref. [6] with $\phi_0(s) \sim \delta_0(s) - \pi$ for $s > 1 \text{ GeV}^2$. The band along each line represents the theoretical uncertainty. The dotted line is the one loop χ PT result [18] and the dot-dashed one the two loop calculation [20].

We show in Fig. 1 our results together with the experimental points from Ref. [19]. The darker band corresponds to employ Ref. [17] for $\delta_0(s)$ below 1 GeV and the lighter one to use Ref. [7]. One sees that now with more precise data one should be able to distinguish between different low energy $\delta_0(s)$ parameterizations as the theoretical uncertainty is much reduced. The widths of the bands correspond to the uncertainties related to the $\delta_0(s)$ and $\delta_2(s)$ parameterizations used, those in fixing the three subtraction constants and in employing Watson's final state theorem for $s > 1 \text{ GeV}^2$, and it also includes the uncertainty in the asymptotic $\phi_I(s)$ employed. In the figure we also show with the dotted line the one loop CHPT result [18] and with the dash-dotted line the two loop one [20]. There is a clear improvement when going from one to two loops in CHPT, though to have a perfect agreement with our results some higher order corrections are still needed. Finally, the dash-double-dotted line corresponds to the result of Ref. [6] with $\phi_0(s) \sim \delta_0(s) - \pi$ for $s > 1 \text{ GeV}^2$. Let us recall that Ref. [6] does not include axial vector exchanges. Were they included, the results of this reference would fall inside the bands shown by our results.

By analytical continuation on the complex plane one can determine the coupling of the σ to $\gamma\gamma$, $g_{\sigma\gamma\gamma}$, and calculate the width to $\gamma\gamma$ of this resonance [2]. We then obtain for the ratio of couplings $\left| \frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}} \right| = (2.1 \pm 0.2) \times 10^{-3}$, with $g_{\sigma\pi\pi}$ the σ coupling to two pions. The result of [6] corresponds to this ratio being 20% bigger at $(2.53 \pm 0.09) \times 10^{-3}$. Half of this difference is due to the omission of the exchanges of axial vector resonances in [6], and the other half comes from improvements delivered by our extra subtraction and our slightly different inputs. As a result, using the same value for $|g_{\sigma\pi\pi}|$ as in [6], our resulting value for $\Gamma(\sigma \rightarrow \gamma\gamma)$ would be around a 40% smaller than that in [6]. Taking into account different choices of $|g_{\sigma\pi\pi}|$ we end with $\Gamma(\sigma \rightarrow \gamma\gamma)$ in the interval $1.8 - 3$ KeV.

4 Conclusions

We have shown that both Ynduráin's method [3] and the solution of the Muskhelishvili-Omnés equations [7, 10] provide compatible results for the quadratic scalar radius of the pion. The origin of the discrepancy between Refs. [3] and [7] was due to overlooking a zero in the scalar form factor in the former reference. We finally obtain [1] $\langle r^2 \rangle_s^\pi = 0.63 \pm 0.05 \text{ fm}^2$ and $\bar{\ell}_4 = 4.5 \pm 0.3$. These numbers are in good agreement with $\langle r^2 \rangle_s^\pi = 0.61 \pm 0.04 \text{ fm}^2$ and $\bar{\ell}_4 = 4.4 \pm 0.2$ of Ref. [7].

We have also studied the $\gamma\gamma \rightarrow \pi^0\pi^0$ reaction for energies $\sqrt{s} \lesssim 0.7 \text{ GeV}$, where S-waves dominate. We have extended the original approach of Ref. [6, 15] by performing a three times subtracted dispersion relation [2], instead of the twice subtracted originally employed. The sensitivity of the results with respect to the phase of the $I = 0$ $\gamma\gamma \rightarrow \pi\pi$ S-wave above $4M_K^2$ is then largely reduced. A key point is to properly handle the contribution of the $f_0(980)$ resonance, at least at the level of the order of magnitude. Importantly, one can then use this reaction to distinguish between different low energy $\pi\pi$ parameterizations once new data on $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ are available. The $\Gamma(\sigma \rightarrow \gamma\gamma)$ width is estimated in the range $1.8 - 3 \text{ KeV}$ [2].

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